

Estimation of actual evapotranspiration and groundwater recharge

Preface

Actual evapotranspiration and groundwater recharge are important terms of the water balance. A method to estimate the average actual evapotranspiration from potential evapotranspiration and soil water supply was presented by Miegel et al.(2013a,b). The method is based on earlier investigations by Glugla and co-workers (Glugla and Tiemer, 1971, Gugla et al., 2003). To perform calculations conveniently, on this website two different computer codes are provided.

Water balance

Without fast surface runoff, long-term groundwater recharge R is given by

$$R = P - E_a \quad (1)$$

P average annual precipitation, cm

E_a average annual average evapotranspiration, cm

Under conditions of hydrologic equilibrium, i.e. $P - E_a - R = 0$, actual evapotranspiration may be calculated implicitly by the Bagrov equation (Glugla et al.1971) given by

$$\frac{dE_a}{dP} = 1 - \left(\frac{E_a}{E_p} \right)^b \quad (2)$$

where E_p denotes potential evapotranspiration. Under wet conditions, E_a/E_p is close to one and the right side of Equ.(2) approaches zero. In this case, actual evapotranspiration does not depend on precipitation. In contrast, under very dry conditions, E_a/E_p is very small and the right hand side of Equ.(2) approaches one. Thus, any change of precipitation yields the same change of actual evapotranspiration or, in other words, the entire precipitation falls prey to evapotranspiration.

Rearranging Equ.(2) leads to

$$P = \int_0^{E_a} \frac{1}{1 - \left(\frac{E_a}{E_p} \right)^b} dE \quad (3)$$

which may be used to calculate E_a . In the computer code described below this is achieved by solving Equ.(3) iteratively until the calculated P equals the observed precipitation.

Estimation of Bagrov coefficient b

The parameter b considers i) the amount of soil water (including capillary rise) available for evapotranspiration and ii) the simultaneity of energy supply and precipitation.

Site conditions are specified by climate, soil hydraulic properties and depth to groundwater. For a range of different soil and climatic conditions, the soil water balance was simulated by the comprehensive numerical model SWAP and an empirical hydro-pedotransfer function was established to estimate the exponent b of Equ.(2) (Miegel et al., 2013). It is given by

$$b = c_1 W_a^{c_2} + c_3 \exp(c_4 q) \quad (4)$$

The hydro-pedotransfer function was incorporated in both of the computer codes GWR.m (Octave-based) and GWR.f (FORTRAN). The variables of this function are described in the following.

Plant available water supply

It is assumed that soil water supply to crops comprises the plant available water of the root zone and the steady-state flow of water from the groundwater table upward to the bottom of the root zone. The first term can be approximated by

$$W_a = d * (\theta(h_{fc}) - \theta(h_{PWP})) \quad (5)$$

W_a	plant-available soil water, cm
d	depth of the root zone, cm
θ	volumetric soil water content
h	soil water pressure head h , positively taken
fc, pwp	field capacity and permanent wilting point, respectively

For soil water retention the vanGenuchten (1980) model is used:

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + (\alpha h)^n)^{(1-1/n)}} \quad (6)$$

where all the items except θ and h are parameters to characterize soil hydraulic properties.

Influence of groundwater table

The so-called capillary rise, i.e. the steady-state flow from the groundwater to the bottom of the root zone is given by the Darcy equation

$$q = -K(h) \left(\frac{dh_p}{dz} + 1 \right) \quad (7)$$

q	flow rate through soil, $\text{cm}^3 \text{cm}^{-2} \text{d}^{-1}$
$K(h)$	soil hydraulic conductivity, cm/d, as function of soil water pressure head h
$h_p = -h$	
z	vertical space coordinate, cm, upward positive

Separation of variables yields with $h=-hp$

$$dz = \frac{dh}{\frac{q}{K(h)} + 1} \quad (8)$$

According to the Mualem/van Genuchten model (vanGenuchten, 1980) $K(h)$ is given by

$$K(h) = K_s \frac{(1 - (\alpha h)^{n-1})(1 + (\alpha h)^n)^{-m}}{(1 + (\alpha h)^n)^{m\tau}} \quad (9)$$

$K(h)$ soil hydraulic conductivity

K_s saturated soil hydraulic conductivity

τ tortuosity parameter

Because of the rather sophisticated form of Equ.(9) the differential equation (8) must be solved numerically.

Simultaneity of water and energy supply

The FORTRAN code described below considers the simultaneity of water and energy availability by a coefficient C_s which may vary from <0.2 for evenly distributed rainfall and evapotranspiration to 1 for extreme nonuniformity. The Octave-based code does not consider C_s .

Users should be aware that this function is very sensitive to local climate conditions. In the region of calibration, summer precipitation (April to September) is about 35 cm, while potential evapotranspiration is approximately 40 cm. If climate conditions are very different and no calibration is intended, it is recommended to set the required coefficient $C_s=0$ as an approximation that should avoid large errors.

Final equation

The final equation is given by $GWR = P - E_a$ (10)

Limitations of the Bagrov method

There are two different conditions where the Bagrov method fails.

(A) Because of the underlying assumption that infiltrated soil water be available to evapotranspiration, the method requires the residence time of infiltrated water in soil to be sufficient to make water available to evapotranspiration.

(B) The second limitation holds for plains under dry climatic conditions where the aquifer is recharged by groundwater inflow from regions with precipitation excess. Since the Bagrov equation restricts actual evapotranspiration to precipitation, it may not be used for wetlands where E_a is enhanced by capillary rise from the groundwater table so much that it might exceed the local precipitation leading to

groundwater depletion. To cope with condition (B), the FORTRAN-based computer program given here uses a statistic-based prediction equation instead of the Bagrov relation. For details, users are referred to Miegel et al.(2013).

Computer code GWR.f

The code is provided as a source file written in FORTRAN and as executable files to run under Windows 7 (*.exe) or Linux (*.go). For the entire input the keyboard is used and results are displayed on the screen. Please note that the unit used throughout the program is centimeter ! The decimal sign is the point. Several data belonging to one prompt must be separated either by a comma or by one or more space characters or by newline. Users are requested to respond to some questions with yes or no. Instead of characters, “0” (zero) is used for “no” and “1” for “yes”.

Input

1. Long-term average annual precipitation and potential evapotranspiration, cm. It is assumed that precipitation data are corrected for measurement errors caused by wind and evaporation from inside the rain gauge. Potential evapotranspiration should be calculated using the Penman-Monteith method.
2. Long-term average annual summer precipitation and potential summer evapotranspiration, cm. In central Europe, the months April to September are considered as summer months.
3. Root zone depth and depth to groundwater table, cm. In this context, the root zone is seen as that part of the soil profile, which is intensely penetrated by plant roots. The soil surface is expected to be plant-covered. The depth to groundwater is considered being constant in time. For that reason, values should apply to those months of the year showing the bigger part of evapotranspiration.
4. Soil class number. A list of texture classes following the german classification is displayed on the monitor. Table 1 provides more information about their average soil properties. Instead of using standard values as offered by the program, different soil hydraulic parameters may be employed (vanGenuchten model). This would also include parameters of capillary rise which must be calculated beforehand. For details see Miegel et. al., 2013. After this input, calculated results on soil hydraulic properties are displayed. These are intended for assessing whether or not estimated soil properties are close to expectations.
5. Coefficient of simultaneity. If this coefficient is not known (- type 0 to indicate this -) monthly values of the average rain and potential evapotranspiration must be entered. The coefficient of simultaneity may vary from <0.2 for evenly distributed rainfall and evapotranspiration to 1 for extreme nonuniformity. As mentioned above, under conditions very different from those of calibration is recommended to enter 1 for knowledge of the coefficient and zero for the coefficient itself.
6. The user is now requested to choose whether or not anaerobiosis is to consider. Anaerobiosis must be expected in wetlands, where almost saturated soils do not exhibit that degree of soil aeration that is necessary for water uptake by plant roots. Since knowledge on anaerobiosis is rather poor and the approximation used here is a mere makeshift users are advised to be very careful in using this option. If on the other hand severe anaerobiosis actually happens then its disregarding would lead to large errors.

Results

Results comprise annual average actual evapotranspiration and groundwater recharge. Investigations (Miegel et al.,2013) have shown that the Bagrov equation predicted the groundwater recharge of the calibration data set with a standard error of RMSE=1.5 cm. If groundwater depletion prevails, this is indicated by a negative recharge value.

We tried to debug the code as far as possible and performed test runs successfully. Nevertheless we disclaim all liability for direct, incidental or consequential damage resulting from the use of the program. Users are advised to compare results with known data of the region considered or with experiences valid for similar site conditions. Please note that the program does not contain any provisions against incorrect input data.

Octave-based code GWR.m

The FORTRAN-based code described above offers the opportunity of using standard values of soil hydraulic properties and selects the appropriate method to calculate groundwater recharge. Its drawback is its complexity or the almost confusing situation users may encounter in case they want to modify the methods provided. For that reason, the same algorithm (although with less services) was again written in OCTAVE, which is a subset of MATLAB. This code was made very clear and evident. Since no compilation step is necessary, users may modify the code according to their needs.

OCTAVEs built-in procedures to solve differential equations (lsode) and to perform numerical integration (quad) shorten the program. It should be easy to incorporate this code into larger problems.

Above all, we hope that users will like the OCTAVE code and appreciate its beauty. We would very much appreciate comments and suggestions to make the code even smarter.

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Table 1: Soil hydraulic parameters of the Mualem/vanGenuchten model valid for German soil texture classes (Renger et al.,2009). Please note that K_0 is a parameter chosen to fit data of unsaturated soil hydraulic conductivity. Parameter m denotes the tortuosity parameter.

The van Genuchten model (van Genuchten, 1980) is given by $\theta = \theta_r + \frac{\theta_s - \theta_r}{(1 + (\alpha h)^n)^{1-1/m}}$

Texture class	Clay %	Silt %	θ_r cm ³ cm ⁻³	θ_0 cm ³ cm ⁻³	α hPa ⁻¹	n 1	τ 1	K_0 cm d ⁻¹
Ss	0-5	0-10	0	0.3879	0.2644	1.3515	-0.59	512
Sl2	5-7	5-20	0	0.3949	0.1165	1.2542	0	192
Sl3	7-12	5-40	0.0519	0.3952	0.07097	1.351	0	90
Sl4	13-17	13-40	0	0.4101	0.1049	1.1843	-3.24	141
Slu	7-15	40-50	0	0.4138	0.08165	1.177	-3.92	109
St2	5-15	0-10	0	0.4049	0.4846	1.1883	-6.19	420
St3	15-25	0-13	0	0.4214	0.1802	1.1323	-3.42	306
Su2	0-5	10-25	0	0.3786	0.2039	1.2347	-3.34	285
Su3	0-7	25-40	0	0.3764	0.08862	1.2140	-3.61	120
Su4	0-7	40-50	0	0.3839	0.3839	1.2223	-3.74	83
Ls2	15-25	40-50	0.1406	0.4148	0.04052	1.3242	-2.07	38
Ls3	15-25	27-40	0.0336	0.4092	0.06835	1.2050	-3.23	98
Ls4	17-20	15-25	0.0463	0.4129	0.09955	1.1821	-3.6	170
Lt2	25-35	35-50	0.149	0.4380	0.07013	1.2457	-3.18	62
Lt3	35-45	30-50	0.1629	0.4530	0.04947	1.1700	-4.10	44
Lts	25-45	17-35	0.1154	0.4325	0.03401	1.1944	0	52
Lu	17-28	50-70	0.0534	0.4284	0.04321	1.1652	-3.23	83
Uu	0-7	80-100	0	0.4030	0.01420	1.2134	-0.56	34
Uls	7-13	50-65	0	0.3985	0.02260	1.1977	-2.04	40
Us	0-7	50-80	0	0.3946	0.02747	1.2239	-2.73	35
Ut2	7-13	>50	0.0101	0.4001	0.01868	1.2207	-1.38	29
Ut3	13-17	>50	0.0053	0.4030	0.01679	1.2067	-1.20	28
Ut4	17-24	>50	0.0276	0.4162	0.01697	1.2048	-0.77	25
Tt	67-100	0-30	0	0.5238	0.06612	1.0522	0	155
Tl	47-67	17-30	0	0.4931	0.07339	1.0625	0	172
Tu2	47-67	>30	0	0.4971	0.07242	1.0606	0	179
Tu3	37-47	>40	0	0.4589	0.0550	1.0817	0	124
Tu4	25-35	>45	0.0170	0.4372	0.04538	1.1204	0	89
Ts2	51-67	0-17	0	0.4836	0.08402	1.0767	0	250
Ts3	35-51	0-17	0.0784	0.4374	0.06194	1.1456	0	118
Ts4	25-35	0-17	0	0.4355	0.2092	1.1142	-7.61	322
fS	0-5	0-10	0	0.4095	0.1504	1.3358	-0.33	285
mS	0-5	0-10	0	0.3886	0.2619	1.3533	-0.58	507
gS	0-5	0-10	0	0.3768	0.2206	1.4657	1.38	872

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