Selected basic tools of soil hydrology written in Octave/Matlab

In general, compiled computer programs run fast and provide detailed and versatile output. Their disadvantage is, that users cannot easily get insight into the functionality of the code. Moreover, it is difficult to modify programs according to special needs of users. Programs written in Octave – Octave is a subset of Matlab – don't need to be compiled beforehand, are visible in every detail and may be customized if necessary. The algorithms of such a code are easy to include into larger and complex programs. Already in former times similar programming tools have been a challenge to treat soil physical problems. For instance, Campbell entitled his book "Soil Physics With Basic" - Basic was a simple programming language in the mid- 20th -century.
As a few samples, we present five small programs, whose names are derived from the main author names of the underlying theory.

1. Haverkamp's set of equations for ponded one-dimensional infiltration in a newer version. Program name: "Haverkamp"
2. Discharge of water from a saturated soil column in contact with groundwater table ("internal drainage"). Name: "Gardner"
3. Discharge of water from a saturated soil column subject to a unit gradient on its bottom boundary. Name: "Sisson"
4. Groundwater recharge dependent on soil, climate, depth to groundwater, rooting depth, and rain distribution. Name: "Bagrov".
5. Parameter optimization of an arbitrary nonlinear function (up to 4 unknown parameters). Name: "Fibonacci"

The codes are written in Octave, running under Ubuntu Linux and are checked as best as we could. Since everything is open to inspection, remaining errors can be detected by the user. The problems one to four are part of the "SHEEP" program on this website and are explained in the description of that program. Since the output is given numerically, it is up to the user to make arrangements for plotting the results. Octave itself provides tools for graphical representation, which are similar to GNUPLOT.

Program "Haverkamp": Ponded infiltration

In 2013, Lassabatere et al. published a new version of Haverkamps infiltration formula. Only unimodal water retention characteristics are considered here. Latorre et al. published in 2018 an investigation on the beta-parameter of that formula. The code presented here is based on both of these publications.

$$\frac{2(\Delta K)^2}{S^2}t=\frac{1}{1-\beta}\left[\frac{2(\Delta K)}{S^2}(I(t)-K_0 t)-\ln\frac{\exp(2\beta(\Delta K/S^2)(I(t)-K_0 t))+\beta-1}{\beta}\right] \quad (1)$$

with

θ_0 initial soil water content

K hydraulic conductivity, D diffusivity

$K_0=K(\theta_0)$

$\Delta K=K_{surface}-K_0$

This equation requires an implicit calculation of cumulative infiltration I.

Sorptivity S is given by

$$S^2=\int_{\theta_0}^{\theta_{surface}}(\theta+\theta_{surf}-2\theta_0)D(\theta)\,d\theta \quad (2)$$

To calculate D and K, the Mualem-vanGenuchten model was assumed:

$$K = K_s S^{\tau}\left[1-\left(1-S^{1/m}\right)^m\right]^2 \qquad D(S) = \frac{K_s S^{\tau-1/m}\left(\frac{1}{A}-2+A\right)}{\left(\theta_s-\theta_r\right)(n-1)\ \alpha} \tag{3,4}$$

References:
New Analytical Model for Cumulative Infiltration into Dual-Permeability Soils
Laurent Lassabatere, Deniz Yilmaz, Xavier Peyrard, Pierre Emmanuel Peyneau, Thomas Lenoir, Jiří Šimůnek, and Rafael Angulo-Jaramillo.-Vadose Zone J. doi:10.2136/vzj2013.10.0181

B. Latorre, D. Moret-Fernández, L. Lassabatere , M. Rahmati, M.V. López ,
R. Angulo-Jaramillo, R. Sorando , F. Comín, J.J. Jiménez:
Influence of the β parameter of the Haverkamp model on the transient soil
water infiltration curve.- Journal of Hydrology 564 (2018) 222–229

Program “Gardner”: Internal drainage

Above groundwater table, a soil column of length L which is initially saturated will approach an equilibrium where the hydraulic gradient vanishes and soil water storage is given by

$$W_{\infty} = \int_{o}^{L} \theta(h(z))\ dz \tag{5}$$

z vertical coordinate, *h* soil water pressure head, θ soil water content

In this state, the discharge will be

$$Q_{\infty} = \theta_s L - W_{\infty} \tag{6}$$

Gardner published in 1962 a solution to the discharge at time t which reads

$$Q_t = Q_{\infty}\left[1-\sum_{j=0}^{j=\infty}\left(\frac{8}{\pi^2(2j+1)}\ \exp\left(-\frac{D_c(2j+1)\ \pi^2 t}{4L^2}\right)\right)\right] \tag{7}$$

Since in this equation diffusivity D_c is a constant, Crank’s method to obtain an effective diffusivity is used:

$$D_C = \frac{1.85}{\left(\theta_i-\theta_f\right)^{1.85}}\int D(\theta)\left(\theta_i-\theta\right)^{0.85} d\theta \tag{8}$$

Because the final water content θ at time t is unknown, an iteration scheme is necessary.

References
Crank, J.: The mathematics of diffusion.-Oxford Science Publications, 1975, 414 pages
Gardner, W.R.: Approximate solution of a non-steady state drainage problem.- Soil Sci. Soc. of America Proceedings 26(1962) 2, 129-132

Program “Sisson”: Unit-gradient discharge from soil

Beneath the soil depth where soil water is available to evapotranspiration and in sufficient large distance to groundwater table the average hydraulic gradient in soil is close to unity. This will cause drainage of soil water with decreasing intensity according to the decreasing of hydraulic conductivity K with decreasing water content θ. To calculate θ(t), an equation proposed by Sisson et al.(1980) may be employed:

$$\frac{dK}{d\theta}=\frac{z}{t} \qquad (9)$$

where t is drainage time and z depth beneath the upper boundary. For the vanGenuchtem/Mualem model of hydraulic properties, the derivative of K with respect to θ is given by

$$\frac{dk}{d\theta}=\frac{K_s}{\theta_s-\theta_r}\left[2S^{\tau+1/m-1}(1-Q^m)Q^{m-1}+\tau S^{(\tau-1)}(1-Q^m)^2\right] \qquad (10)$$

with

$$Q=1-S^{(1/m)} \qquad (11)$$

S is the effective saturation $S=(\theta-\theta_r)/(\theta_s-\theta_r)$
To make calculations convenient, Equ. (10) was rearranged.
From Equs.(9 and 10) the water content θ may be calculated implicitly. Integration of θ yields water storage in soil.

Reference
Sisson, J.B.,Ferguson, A.H.,v.Genuchten, M.TH.
Simple method for predicting drainage from field plots.-Soil Science Society of Am. Journal 44(1980) 1147-1152

Program “Bagrov”: Groundwater recharge dependent on soil, climate, depth to groundwater, rooting depth, and rain distribution.

Water balance

Without fast surface runoff, long-term groundwater recharge R is given by

$$R = P - E_a \qquad (12)$$

P average annual precipitation, cm
E_a average annual average evapotranspiration, cm

Under conditions of hydrologic equilibrium, i.e. $P-E_a-R=0$, actual evapotranspiration may be calculated implicitly by the Bagrov equation (Glugla et al.1971) given by

$$\frac{dE_a}{dP}=1-\left(\frac{E_a}{E_p}\right)^b \qquad (13)$$

where E_p denotes potential evapotranspiration. Under wet conditions, E_a/E_p is close to one and the right side of Equ.(2) approaches zero. In this case, actual evapotranspiration does not depend on precipitation. In contrast, under very dry conditions, E_a/E_p is very small and the right hand side of Equ.(2) approaches one. Thus, any change of precipitation yields the same change of actual evapotranspiration or, in other words, the entire precipitation falls prey to evapotranspiration.

Rearranging Equ.(2) leads to

$$P=\int_0^{E_a} \frac{1}{1-\left(\frac{E_a}{E_p}\right)^b} dE \tag{14}$$

which may be used to calculate E_a.

Estimation of Bagrov coefficient b

The parameter b considers i) the amount of soil water (including capillary rise) available for evapotranspiration and ii) the simultaneity of energy supply and precipitation.

Site conditions are specified by climate, soil hydraulic properties and depth to groundwater. For a range of different soil and climatic conditions, the soil water balance was simulated by the comprehensive numerical model SWAP and an empirical hydro-pedotransfer function was established to estimate the exponent b of Equ.(2) (Miegel et al., 2013). It is given by

$$b=c_1 W_a^{c2}+c_3(\exp(c_4 q)-1) \tag{15}$$

Plant available water supply

It is assumed that soil water supply to crops comprises the plant available water of the root zone and the steady-state flow of water from the groundwater table upward to the bottom of the root zone. The first term can be approximated by

$$W_a=d*(\theta(h_{fc})-\theta(h_{PWP})) \tag{16}$$

W_a	plant-available soil water, cm
d	depth of the root zone,cm
θ	volumetric soil water content
h	soil water pessure head h, positively taken
fc, pwp	field capacity and permanent wilting point, respectively

For soil water retention the vanGenuchten (1980) model is used:

$$\theta(h)=\theta_r+\frac{\theta_s-\theta_r}{\left(1+(\alpha h)^n\right)^{(1-1/n)}} \tag{17}$$

where all the items except θ and h are parameters to characterize soil hydraulic properties.

Influence of groundwater table

The so-called capillary rise, i.e. the staedy-state flow from the groundwater to the bottom of the root zone is given by the Darcy equation

$$q=-K(h)\left(\frac{dhp}{dz}+1\right) \tag{18}$$

q flow rate through soil, $cm^3cm^{-2}d^{-1}$
K(h) soil hydraulic conductivity, cm/d, as function of soil water pressure head h
hp=-h
z vertical space coordinate, cm, upward positive

Separation of variables yields with h=-hp

$$dz=\frac{dh}{\frac{q}{K(h)}+1} \tag{19}$$

According to the Mualem/van Genuchten model (vanGenuchten, 1980) K(h) is given by

$$K(h)=K_s\frac{\left(1-(\alpha h)^{(n-1)}\left(1+(\alpha h)^n\right)^{-m}\right)^2}{\left(1+(\alpha h)^n\right)^{m\tau}} \tag{20}$$

K(h) soil hydraulic conductivity
K_s saturated soil hydraullic conductivity
τ tortuosity parameter

Because of the rather sophisticated form of Equ.(9) the differential equation (8) must be solved numerically.

The final equation is given by $GWR=P-E_a$

Limitations of the Bagrov method

There are two different conditions where the Bagrov method fails.
(A) Because of the underlying assumption that infiltrated soil water be available to evapotranspiration, the method requires the residence time of infiltrated water in soil to be sufficient to make water available to evapotranspiration.
(B) The second limitation holds for plains under dry climatic conditions where the aquifer is recharged by groundwater inflow from regions with precipitation excess. Since the Bagrov equation restricts actual evapotranspiration to precipitation, it may not be used for wetlands where E_a is enhanced by capillary rise from the groundwater table so much that it might exceed the local precipitation leading to groundwater depletion. To cope with condition (B), the FORTRAN-based computer program given here uses a statistic-based prediction equation instead of the Bagrov relation. For details, users are referred to Miegel et al.(2013).

References

K. Miegel, K. Bohne, and G. Wessolek
Prediction of long-term groundwater recharge by using hydropedotransfer functions .-Int. Agrophys., 2013, 27, doi: 10.2478/v10247-012-0013-y

Miegel, K., K. Bohne, G.Wessolek
Vorhersage der regionalen, langjährigen bodenbürtigen Grundwasserneubildung mit hydrologischen Pedotransferfunktionen.- Hydrologie und Wasserbewirtschaftung Band 57,Heft 6, 2013

Program “Fibonacci”: Parameter optimization of an arbitrary nonlinear function

Many highly sophisticated tools are available to fit nonlinear equations to measured data. Sometimes a simple and robust tool is needed to fit equations without using the Excel solver or MatCad or gnuplot or RETC in case of water retention data. The program presented here is based upon certain numbers mentioned already by Leonardo da Pisa (called Fibonacci) in 1202. The method assumes, that the objective function, measuring the goodness of fit, has no more then one minimum within a current search interval. Based on this assumption, we can conclude, that the minimum cannot lie outside the search border attributed with the larger error. In this way it is possible to scale down the search interval step by step.

The program presented here requires knowledge of reasonable initial estimates. If the position of the global error minimum is very obscure, a Monte-Carlo search step should be performed beforehand. This is not included in the Octave file but incorporated in the FORTRAN version on this website. The program comes with an example and test data, that is the van Genuchten water retention function.

The test data are generated by the vanGenuchten function using assumed parameters. If the program runs correct, the estimated parameters should be very close to the assumed ones. Users may try any other function of their demand.

References

Leonardo da Pisa: Liber Abaci, Pisa 1202
Vardavas, I.M. (1989): A Fibonacci search technique for model parameter selection.
Ecological Modeling 48, 65-81, Amsterdam